A comparative study on value at risk and conditional value at risk with an application to the Malaysian financial market

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A comparative study on value at risk and conditional value at risk with an application to the Malaysian financial market

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Value at risk (VaR) and conditional value at risk (CVaR) are frequently used as risk measures in risk management. VaR estimates the maximum expected loss over a given time period at a given acceptance level, whereas CVaR measures the extreme risk or the risk beyond VaR. This paper aims to perform an empirical study on VaR and CVaR based on the daily returns of the Malaysian stock markets traded in Kuala Lumpur Composite Index (KLCI) over a time period using the RiskMetrics and the peaks over the threshold (POT) methods. In particular, the IGARCH (1, 1) model is applied for the RiskMetrics method, whereas the generalized Pareto distribution (GPD), a distribution based on an extreme value theory, is considered for the POT method. The results show that the GPD, which is considered in the POT method, provides an adequate fit to the data of threshold excesses, and the POT is a more reliable measure of risks compared to the RiskMetrics.

Key words: Value at risk, conditional value at risk, RiskMetrics, peaks the over threshold, IGARCH, generalized Pareto.

INTRODUCTION

Value at risk (VaR) is widely used in risk management. Historically, the concept of VaR is related to the covariance method that was first adopted by J. P. Morgan Bank as a branch standard, called a RiskMetrics model. Since the Group of Thirty report in 1996, VaR has become the corner-stone in the risk management framework and is essential in allocating a capital as a cushion for market risk exposures. While primarily designed for market risk exposures, VaR methodology now underpins the credit and the operational risk recommendations. Based on the internal model approach endorsed by the Basel Committee on Banking and Supervision of Banks for Internal Settlement (Basel Committee on Banking Supervision, 1995) and later adopted by the US bank regulators, banks are allowed to use their own models to estimate VaR and use it as one of the guidelines to keep aside regulatory capitals.

From a statistical viewpoint, VaR is a given percentile of the profit (or loss) distribution over a fixed time horizon. To be acceptable by regulators, the confidence level must be 99% and the holding period must be two weeks, or equivalently, ten trading days. This is motivated by the fear of a liquidity crisis where a financial institution might not be able to liquidate its holdings for ten straight days. However, market participants consider the 99% confidence level and the two weeks horizon to be too conservative. As an additional tool for internal risk controlling, both the holding period and the confidence level can be selected to fit the needs of the analysts. In practice, it is common to limit the confidence level to 95% and the holding period to one day.

The existing approaches for estimating the profit (or loss) distribution can be divided into three main groups: the non-parametric historical simulation method; the fully parametric methods based on an econometric model for volatility dynamics and the assumption of conditional distribution such as GARCH models; and the methods based on an extreme value theory. This study focuses on

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the second approach, that is, the fully parametric method, and the third approach, that is, the method based on an extreme value theory.

Within the fully parametric literature, many researches has either forecasted VaR at different time horizons or applied VaR to assess the performance of a particular model. Therefore, the contributions of this paper to the existing literature are as follows: firstly, the study provides a manner to approximate the day-ahead VaR density when the underlying process is described by an econometric model (GARCH model) and the peaks over the threshold (POT) approach. Secondly, it extends the methodology to the conditional VaR, or the CVaR, and show that the procedure can also be applied in a straightforward manner. Lastly, this paper aims to provide a greater understanding of VaR and CVaR modelling procedures, as well as the risks of share prices, in the Malaysian context. The application of several VaR metrics should indicate whether the measures are robust and consistent over time and thus, providing a rational justification to the choice of a point estimate. Therefore, the results of this study can be used by investors to select their optimal point estimate of VaR, and by the regulatory body to measure regulatory capitals.

The layout of this paper is as follows: definition of the concept of VaR and the derivation of CVaR; review of the fundamentals of the RiskMetrics and POT methods; an empirical application of VaR and CVaR on the Malaysian financial data; and finally, the conclusions.

RISK MEASURES

Traditional value at risk (VaR)

Value at risk (VaR) is a procedure designed to forecast the maximum expected loss over a target horizon, given a statistical confidence limit. Nevertheless, despite its popularity, VaR has certain undesirable mathematical properties such as lack of sub-additivity and convexity (Arztner et al., 1999, 1997; Alexander and Leigh, 1997; Benninga and Wiener, 1998). In the case of a standard normal distribution, VaR is proportional to the standard deviation and is coherent when based on this distribution but not in other circumstances. In addition, the VaR resulting from a combination of two portfolios can be greater than the sum of risks of individual portfolios (Pritsker, 1997; Duffie and Pan, 1997). A further compli-cation is associated with the fact that VaR is difficult to be optimized when calculated from scenarios (Mckay and Keefer, 1996; Mauser and Rosen, 1999). VaR relies on a linear approximation of the portfolio risks and assumes a joint normal (or log-normal) distribution of the underlying market processes.

Suppose that at the time index t, we are interested in the risk of a financial position for the next k periods. Let $\Delta t = k$ be the change in time and $\Delta V(k)$ be the change

in the value of assets in a financial position from time t to $t+k$ where this quantity is measured in a currency and is a random variable at the time index t, and $L(k)$ be the associated loss function. $L(k)$ is a positive or negative function of $\Delta V(k)$ depending on the position being short or long. Let $F_k(x)$ denotes the cumulative distribution function (CDF) of $L(k)$. The VaR of a financial position over the time horizon k with tail probability p can be defined as,

$$p = \Pr(L(k) \geq VaR) = 1 - F_k(VaR) \qquad (1)$$

Since the holder of a long financial position suffers a loss when $L(k) < 0$, VaR typically assumes a negative value when p is small. Therefore, a negative sign signifies a loss. From this definition, the probability that the holder would encounter a loss greater than or equal to VaR over the time horizon k is p. Alternatively, VaR can be interpreted as follows: with probability $1-p$, the potential loss encountered by the holder of the financial position over the time horizon k is less than or equal to VaR. If $F_k(x)$ is known, then VaR is simply the $(1-p)$th quantile of the CDF of the loss function $L(k)$.

Conditional value at risk (CVaR)

Conditional value at risk (CVaR) measures the extreme risk or the risk beyond VaR. CVaR is also called the mean excess loss and is a coherent risk measure that has many attractive properties (Alexander and Baptista, 2003; Ogryczak and Ruszczynski, 2002; Rockafellar and Uryasev, 2002; Pflug, 2000). When an extreme loss occurs or equivalently when VaR is exceeded, the actual loss can be much higher than VaR. To better quantify the loss and to employ a more coherent risk measure, the expected loss once VaR is exceeded should be considered. CVaR is defined as:

$$CVaR_q = E(L(k) \mid L(k) > VaR_q) \qquad (2)$$

where $q = 1-p$.

MATERIALS AND METHODS

The dataset for the study is obtained from the datastream which provides information on the daily returns of stock markets traded in Kuala Lumpur Composite Index (KLCI). In particular, VaR and CVAR are studied and compared on the following sectors: trading-services, finance, property, consumer-products, industrial-products, infrastructure, construction, technology, plantation and hotel.

VaR and CVaR models

RiskMetrics

Morgan (1996) developed the RiskMetrics[TM] methodology to calculate VaR (Hull and White, 1998; Raaji and Raunig, 1998; Hamilton, 1994). Let r_t denotes the daily log return which is measured in a percentage, and F_{t-1} the information set available at time $t-1$. The RiskMetrics assume that $r_t \mid F_t \sim N(0, \sigma_t^2)$, where the conditional variance of r_t, σ_t^2, follows the special IGARCH (1, 1) model:

$$\sigma_t^2 = \alpha \sigma_{t-1}^2 + (1-\alpha) r_{t-1}^2, \quad 1 > \alpha > 0. \tag{3}$$

where α is the IGARCH coefficient. Under the special IGARCH (1, 1) model, the conditional distribution, $r_t(k) \mid F_t$, is normally distributed with mean zero and variance $\sigma_t^2(k)$.

By using the independence assumption, we have:

$$\sigma_t^2(k) = Var(r_t(k) \mid F_t) = \sum_{i=1}^{k} Var(a_{t+i} \mid F_t),$$

where $Var(a_{t+i} \mid F_t) = E(\sigma_{t+i}^2 \mid F_t)$ can be obtained recursively. Using $r_{t-1} = a_{t-1} = \sigma_{t-1} \varepsilon_{t-1}$, where ε is the standard Gaussian white noise series, we can rewrite the model as:

$$\sigma_t^2 = \sigma_{t-1}^2 + (1-\alpha)\sigma_{t-1}^2(\varepsilon_{t-1}^2 - 1). \tag{4}$$

Therefore, VaR and CVaR can be calculated as:

$$VaR_q = \Phi^{-1}(\alpha)\sigma_t \tag{5}$$

and

$$CVaR_q = \frac{f(VaR_q)}{p} \times \sigma_t \tag{6}$$

where $\Phi(.)$ is the CDF and $f(.)$ is the probability density function (PDF) of a standard normal distribution.

Peaks over the threshold (POT)

Instead of focusing on the extremes (maximum or minimum), the POT approach focuses on the measurement of the exceedances over a certain high threshold and the times at which the exceedances occur (Neftci, 2000; Danielsson and deVries, 1997; Davison and Smith, 1990; Smith, 1989).

Suppose T is a sample size and β, α and k are the location, the scale and the shape parameters of a distribution obtained from an extreme value theory. For a given tail probability p, where $q = 1 - p$, the VaR can be defined as:

$$VaR = \begin{cases} \beta - \dfrac{\alpha}{k}\{1 - [-T\ln(q)]^{-k}\} & k \neq 0 \\ \beta - \alpha \ln[-T\ln(q)], & k = 0 \end{cases} \tag{7}$$

For simplicity, assume that the shape parameter is a non-zero value, $k \neq 0$. Consider an extreme value distribution such as a generalized Pareto distribution (GPD) where η is the threshold, k is the shape parameter and $\psi(\eta) = \alpha + k(\eta - \beta)$ is the scale parameter. For a given threshold η, the parameters, k and $\psi(\eta)$, can be obtained by fitting the GPD to the sample data. Therefore, VaR can be computed as:

$$VaR_q = \eta - \frac{\psi(\eta)}{k}\left(1 - \left(\frac{T}{N_\eta}(1-q)\right)^{-k}\right) \tag{8}$$

which depends on η, and N_η is the number of exceedances of the threshold η. CVaR measures the loss given that the VaR is exceeded. Specifically,

$$\begin{aligned} CVaR_q &= E(r \mid r > VaR_q) \\ &= VaR_q + E(r - VaR_q \mid r > VaR_q) \end{aligned} \tag{9}$$

Therefore, for the GPD distribution, Equation 9 can be rewritten as,

$$CVaR_q = \frac{VaR_q}{1-k} + \frac{\psi(\eta) - k\eta}{1-k}. \tag{10}$$

RESULT AND DISCUSSION

The daily return series, r_t, is given by the log difference of the closing price, P_t, which is measured in Ringgit Malaysia (RM) currency and computed as $R_t = \ln(P_t) - \ln(P_{t-1})$. It should be noted that the currency of Ringgit Malaysia (RM) was pegged at RM3.80 = USD1 on 2 September 1998 and shifted to a managed float against a basket of currencies as of 21 July 2005. To investigate the performance of VaR and CVaR models, we use the in-sample evaluation periods, which span a period of approximately 14 years from 19 December 1996 to 6 August 2010.

Table 1 displays the summary statistics of the equity market returns. It can be seen that seven out of ten sectors have positive mean returns. The property, technology and hotel sectors display negative means, which could be explained by the effect of the recent credit crunch financial crisis in 2008. Surprisingly, the financial

Table 1. Summary statistics for daily equity market returns in KLCI.

Sector	Mean	Std. dev.	Skewness	Kurtosis	Q(10)	Q^2(10)	ARCH(10)
Trading/services	0.008	0.681	-0.109	4.401	39.934**	83.924**	71.1418**
Finance	0.023	1.220	-0.443	22.798	122.464**	37.140**	30.3614**
Property	-0.006	1.804	0.507	7.344	43.778**	433.100**	317.716**
Consumer/products	0.029	1.113	-0.595	12.073	33.960**	349.438**	280.961**
Industrial/products	0.016	0.974	0.384	7.745	28.902**	166.945**	113.589**
Infrastructure	0.016	2.752	0.172	48.450	64.355**	741.126**	558.931**
Construction	0.048	1.769	-0.070	7.479	28.028**	594.140**	322.258**
Technology	-0.022	2.556	-0.364	12.232	22.087**	373.106**	283.970**
Plantation	0.007	1.970	0.193	11.573	32.754**	848.616**	438.931**
Hotel	-0.017	2.330	0.540	11.127	35.759**	894.125**	519.408**

** and * are statistically significant at 1 and 5%, respectively.

sector has a positive mean. However, this sector has a negative skewness; implying that the distribution of the log-returns tends to be on a negative side, or in other words, the financial sector generally has more losses than gains. The mean returns of all ten sectors are generally close to zero, characterized by high volatilities.

The log-returns are also leptokurtic in nature, with higher peaks and fatter tails compared to the normal distribution. In general, the normal skewness (0.00) and the normal excess kurtosis (3.00) are rejected at a 5% significance level.

The Ljung-Box Q-statistics, specifically the Q(10) and the Q^2(10) under the null hypothesis of non-serial correlation tests in daily returns and squared returns, are also displayed in Table 1. At a significance level of 5%, the null hypothesis is rejected. The LM test is highly significant, which is an evidence of the presence of an ARCH effect, indicating the legitimacy of using the ARCH or the GARCH models (Bollerslev, 1986; Engle, 1982). We may conclude that all series show departures from the normality and the serially correlated assumptions. The presence of several kurtosis predicts the legitimacy of using a fatter-tailed distribution such as the student-t rather than the normal distribution.

Figure 1 looks at the returns behaviour of the ten sectors traded in KLCI over the sample period. There is an evidence of volatility clustering where large or small price changes tends to be followed by other large or small price changes of either sign, positive or negative. This implies that the log return volatility changes over time. Figure 1 also shows the mean excess plot of the negative returns of the ten sectors, where an upward sloping plot indicates a heavy-tailed behaviour. In particular, a straight line with a positive slope is a sign of Pareto behaviour in the tail.

For risk management purpose, the investors may be interested in the frequency of the occurrence of large losses above a certain high threshold as well as the average value of losses that exceed the high threshold, that is, they may be interested in the daily VaR and CVaR. The mean excess modelling for extreme values above a high threshold may be used to address these issues.

Table 2 reports the parameter estimates and the residuals diagnostic generated from the IGARCH (1, 1) model of the RiskMetrics method. As expected, based on the Q-statistics, the model is rejected for most sectors with the exception to the finance, infrastructure and technology sectors. In particular, the study obtains a highly significant statistic for Q^2(15) of the squared standardized residuals.

Table 3 display VaR and CVaR values computed from the RiskMetrics and the POT models. For the RiskMetrics model, it can be seen that the daily returns, which are provided by the VaR values with 5% probability, are as low as -0.013, -0.014, -0.026, -0.019, -0.012, -0.059, -0.030%, -0.018, -0.030 and -0.072%, respectively for the trading-services, finance, property, consumer-products, industrial-products, infrastructure, construction, techno-logy, plantation and hotel sectors. The average returns, which are provided by the CVaR values, are 0.010, 0.012, 0.041, 0.021, 0.008, 0.295, 0.056, 0.029, 0.055 and 0.319%, respectively for the same sectors. As an example, if one holds a long position on the stock of the Trading-Services sector worth RM10 million, then the estimated VaR with tail probability 5% is –RM126,000. The estimated CVaR, which indicates the average returns or the expected shortfall (ES) associated with the given VaR for the same stock position and the same sector is RM97,000.

The estimated VaR and CVar values for the POT model are also displayed for a given tail probability of 5%, assuming that the number of exceedances is ten. It can be concluded by comparing the RiskMetrics and the POT models, that the RiskMetrics underestimates the risks, or understates the risk coverage probabilities of most sectors.

Figures 2 to 11 shows that the diagnostic plots of the GPD fitted to the daily negative returns of the ten sectors traded in KLCI. The QQ-plots (lower-right panel) and the
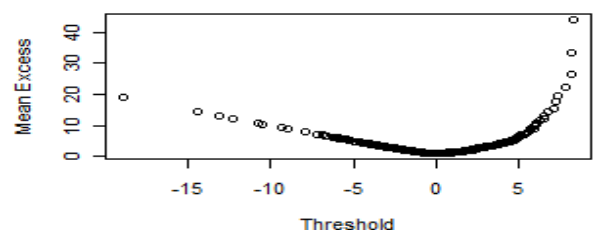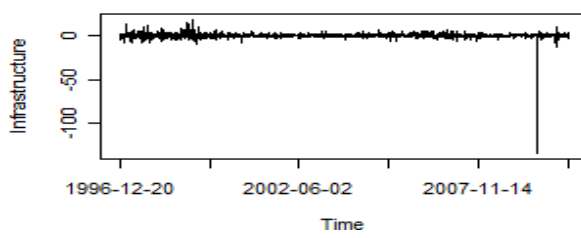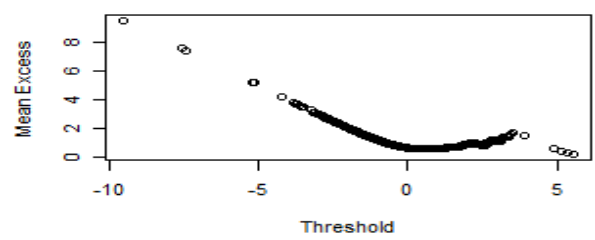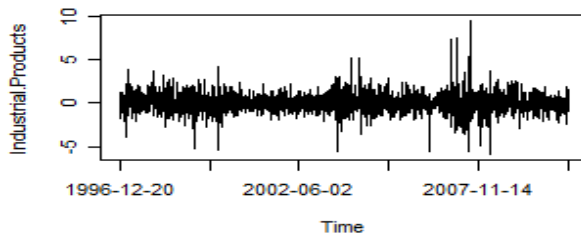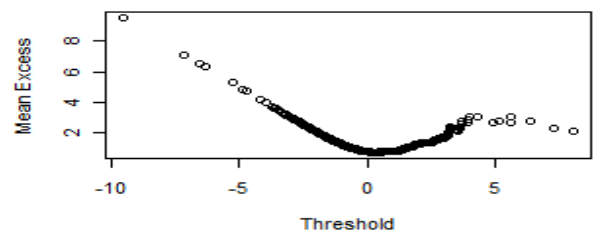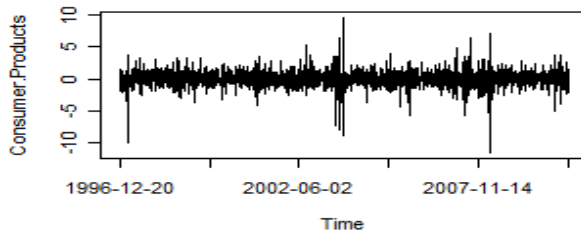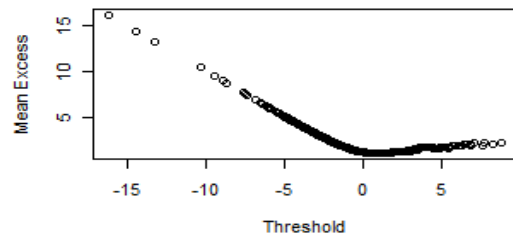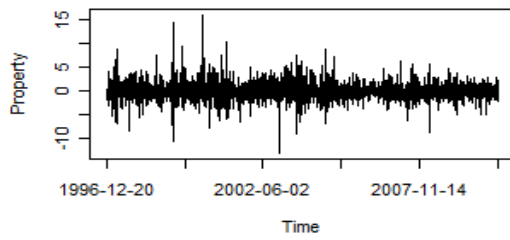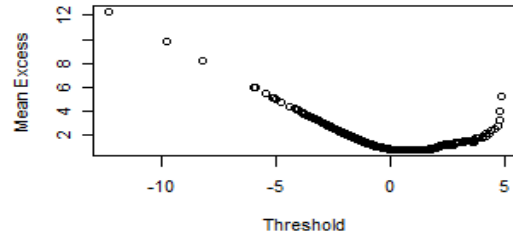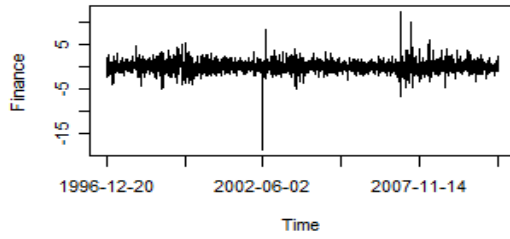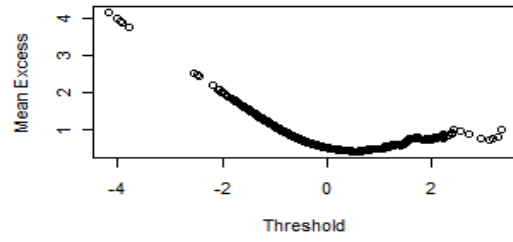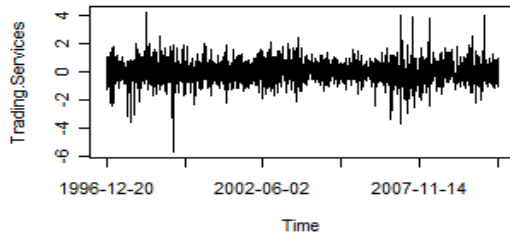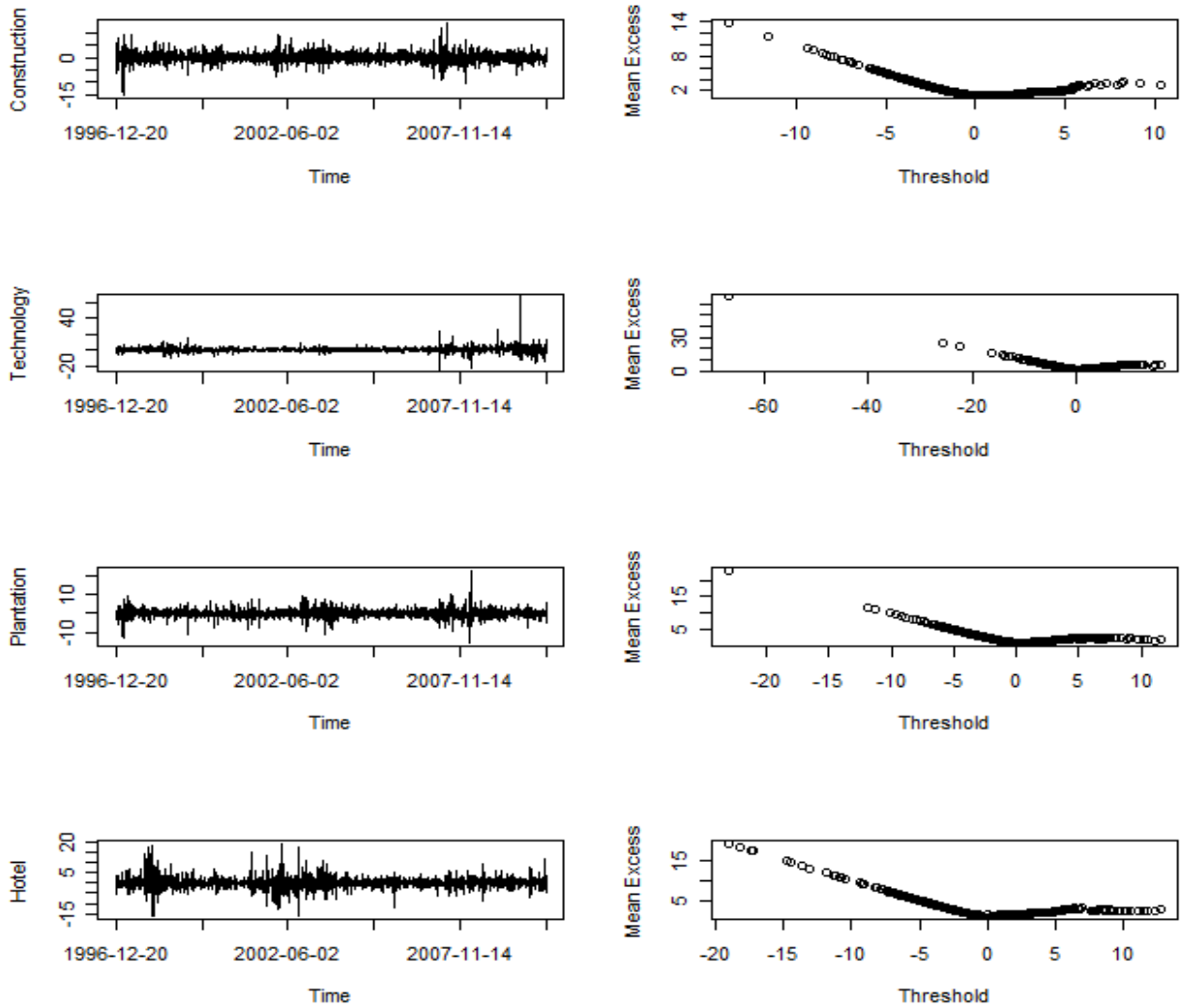
Figure 1. Daily log return and mean excess return for 10 sectors in KLCI.

**Table 2.** Parameter estimates and diagnostics of IGARCH (1, 1) model.

Sector	W_0	α	$1-\alpha$	Q(10)	Q^2(10)	ARCH(10)	AIC
Trading/services	0.0120	0.0183 (0.0000)	0.9819	36.1042 (0.0017)	29.2769 (0.006)	1.9608 (0.0335)	2.0354
Finance	0.0533	0.0385 (0.0000)	0.9617	79.4254 (0.0000)	0.4021 (1.0000)	0.0220 (1.0000)	3.2027
Property	-0.0110	0.0598 (0.0000)	0.9404	40.4679 (0.0004)	30.0630 (0.0046)	2.7531 (0.0022)	3.9038
Consumer/products	0.0560	0.0487 (0.0000)	0.9515	21.7049 (0.1158)	28.3104 (0.0082)	2.4324 (0.0069)	2.9182
Industrial/products	0.0199	0.0315 (0.0000)	0.9687	33.3096 (0.0043)	22.4705 (0.0485)	2.0405 (0.0260)	2.6690
Infrastructure	0.0707	0.0219 (0.0000)	0.9782	21.2911 (0.1278)	0.5698 (0.0000)	5.6344 (0.0000)	4.2615
Construction	0.0661	0.0413 (0.0000)	0.9589	18.0011 (0.2626)	40.7989 (0.0001)	3.4833 (0.0001)	3.8350
Technology	-0.0468	0.0479 (0.0000)	0.9523	18.0575 (0.2596)	16.6318 (0.21667	1.4626 (0.1469)	4.4331
Plantation	0.0295	0.0524 (0.0000)	0.9478	23.4993 (0.0741)	28.7319 (0.0071)	2.3193 (0.0102)	3.9519
Hotel	0.0041	0.0487 (0.0000)	0.9515	39.3856 (0.0006)	47.8568 (0.0000)	4.3488 (0.0000)	4.2160

tail probability estimates (in log scale at the upper-right panel) show several minor deviations from a straight line for several sectors. In general, the GPD, which is used in the POT method, appears to provide a fairly well fit to the

Table 3. Predictive quantile loss at 5%.

Sector	RiskMetrics		POT	
	VaR	CVaR	VaR	CVaR
Trading/services	-0.0126	0.0097	-1.7992	-0.1897
Finance	-0.0143	0.0124	3.1867	3.5340
Property	-0.0258	0.0406	-2.7578	0.7817
Consumer/products	-0.0187	0.0213	-17.7587	-8.4371
Industrial/products	-0.0115	0.0080	-34.6046	-15.0109
Infrastructure	-0.0586	0.2952	5.1492	7.7047
Construction	-0.0303	0.0557	-16.3866	-6.7000
Technology	-0.0183	0.0289	-13.3738	-2.8679
Plantation	-0.0300	0.0547	-3.4542	1.4896
Hotel	-0.0724	0.3186	0.1923	3.9337
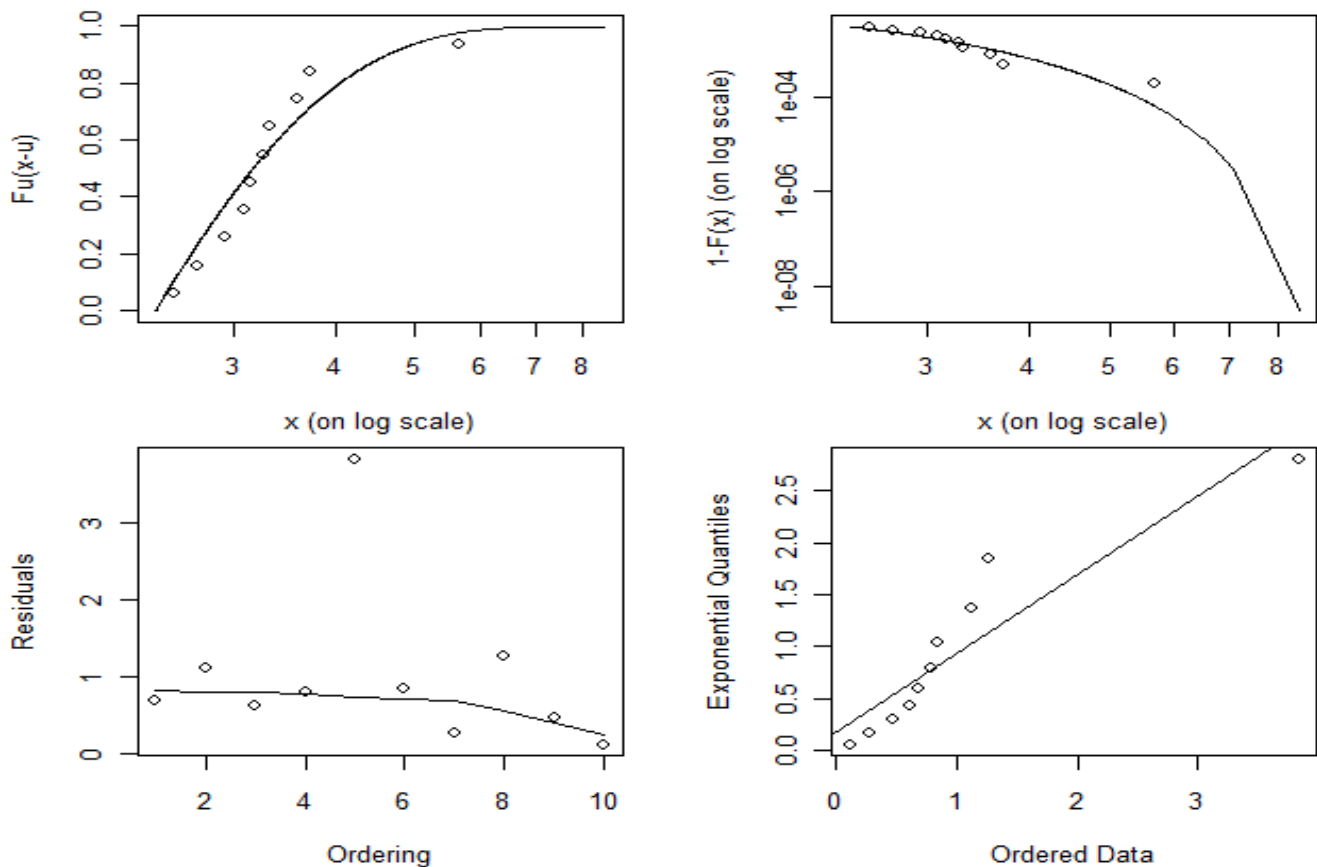
Figure 2. Diagnostic plots for trading/services sector.

data of threshold excesses

Conclusions

This paper has performed a comparative study on VaR and CVaR based on the daily returns of the Malaysian share markets traded in Kuala Lumpur Composite Index (KLCI) over a time period. The RiskMetrics method with the IGARCH (1,1) model, and the POT method with the GPD are applied to calculate VaR and CVaR of ten industrial sectors in Malaysia, namely trading-services,
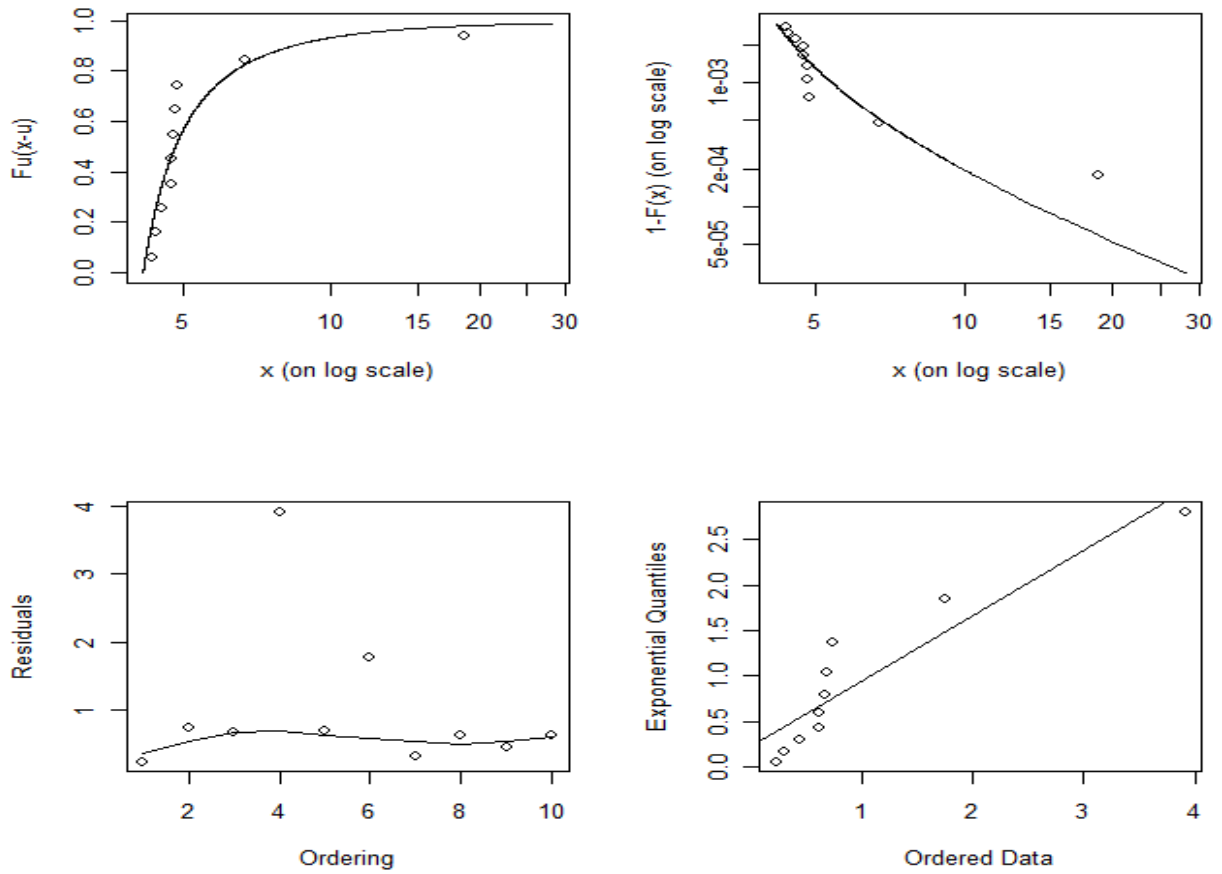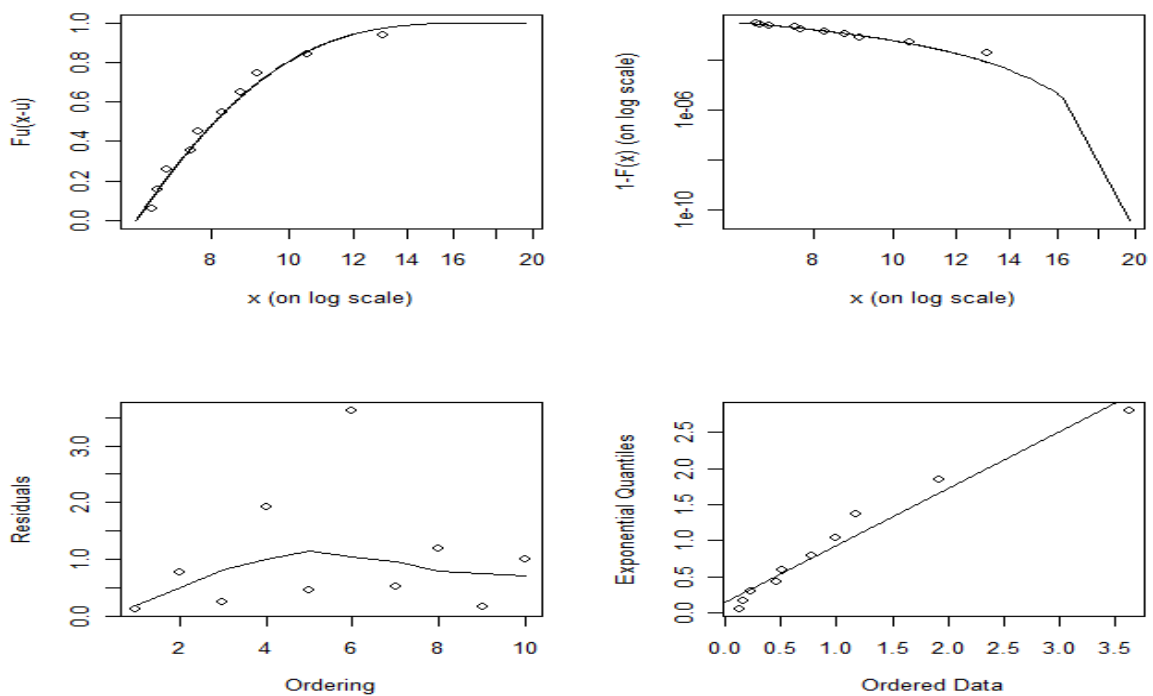
Figure 3. Diagnostic plots for finance.

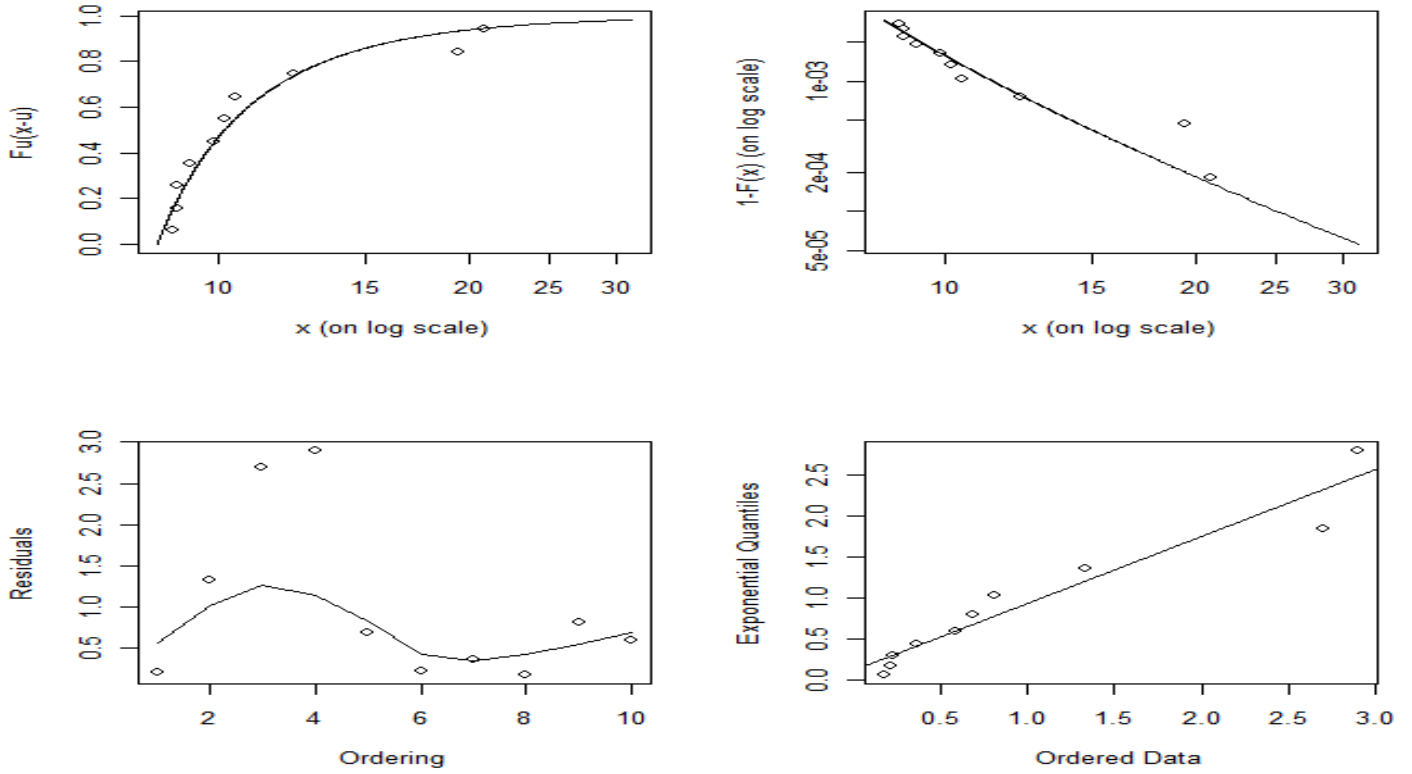**Figure 4.** Diagnostic plots for property.

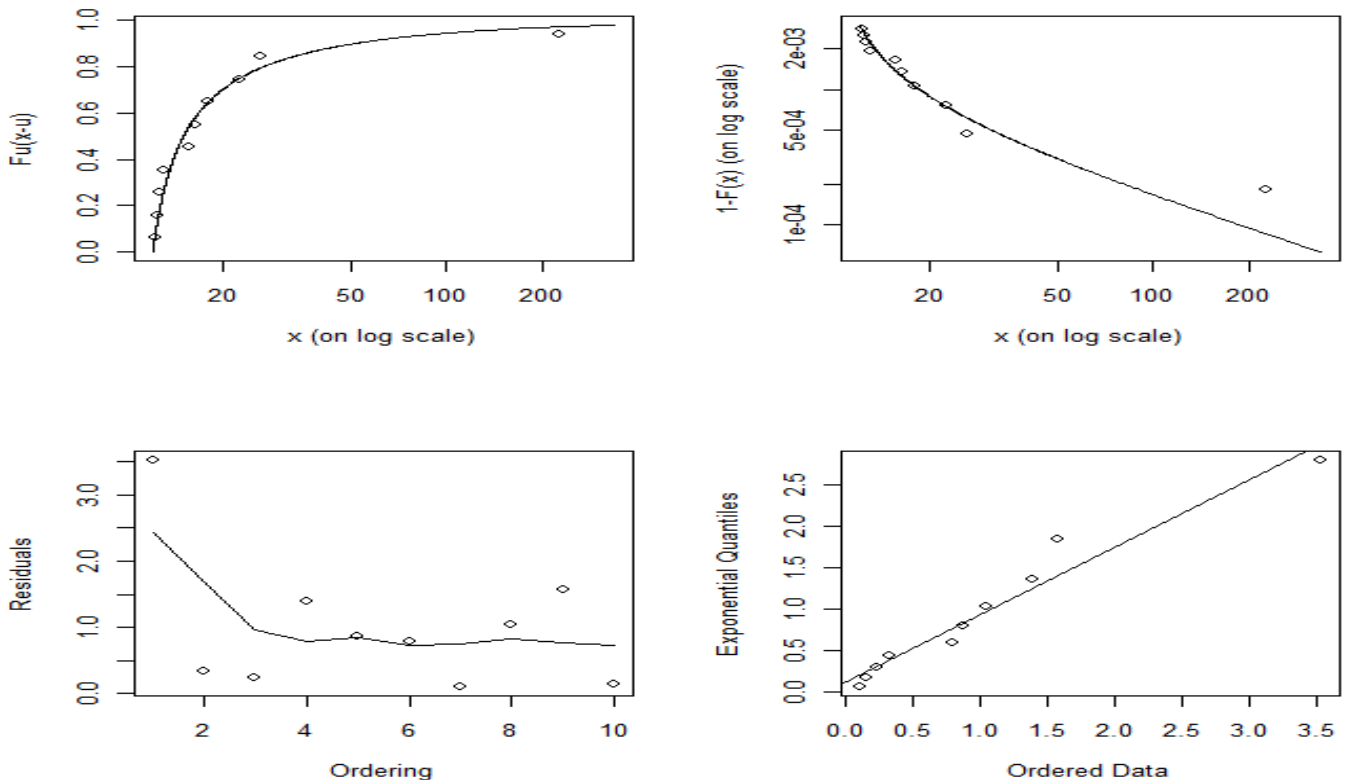**Figure 5.** Diagnostic plots for consumer/products sector.

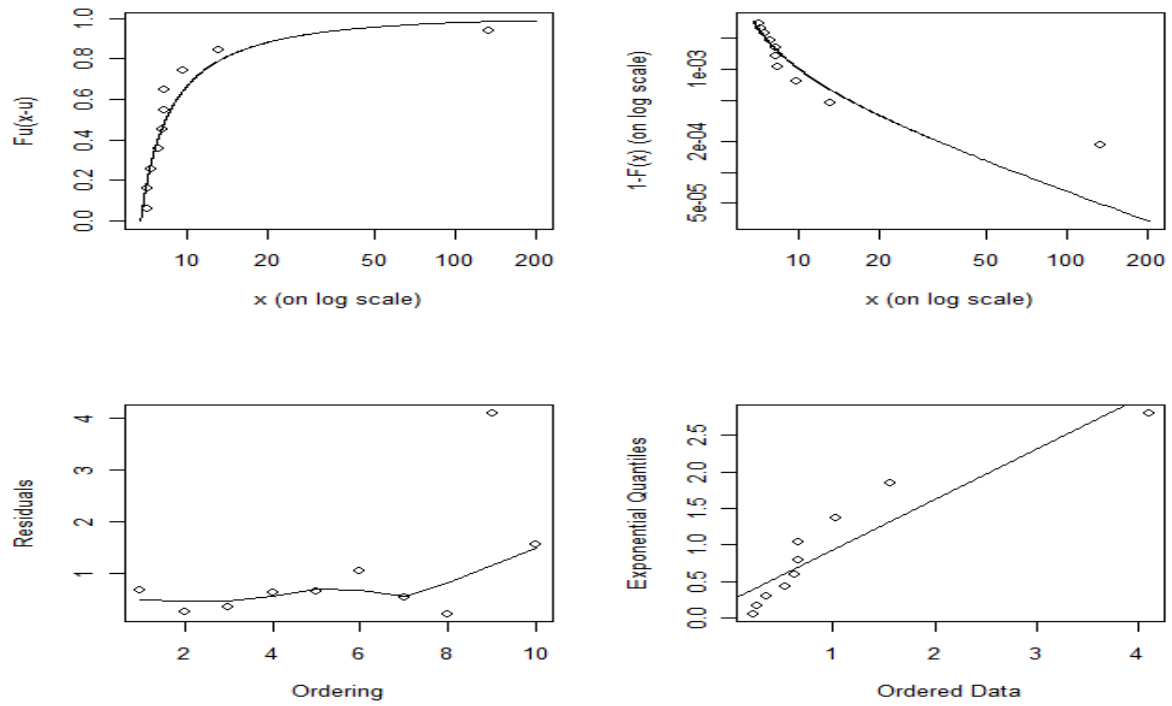**Figure 6.** Diagnostic plots for industrial/products sector.

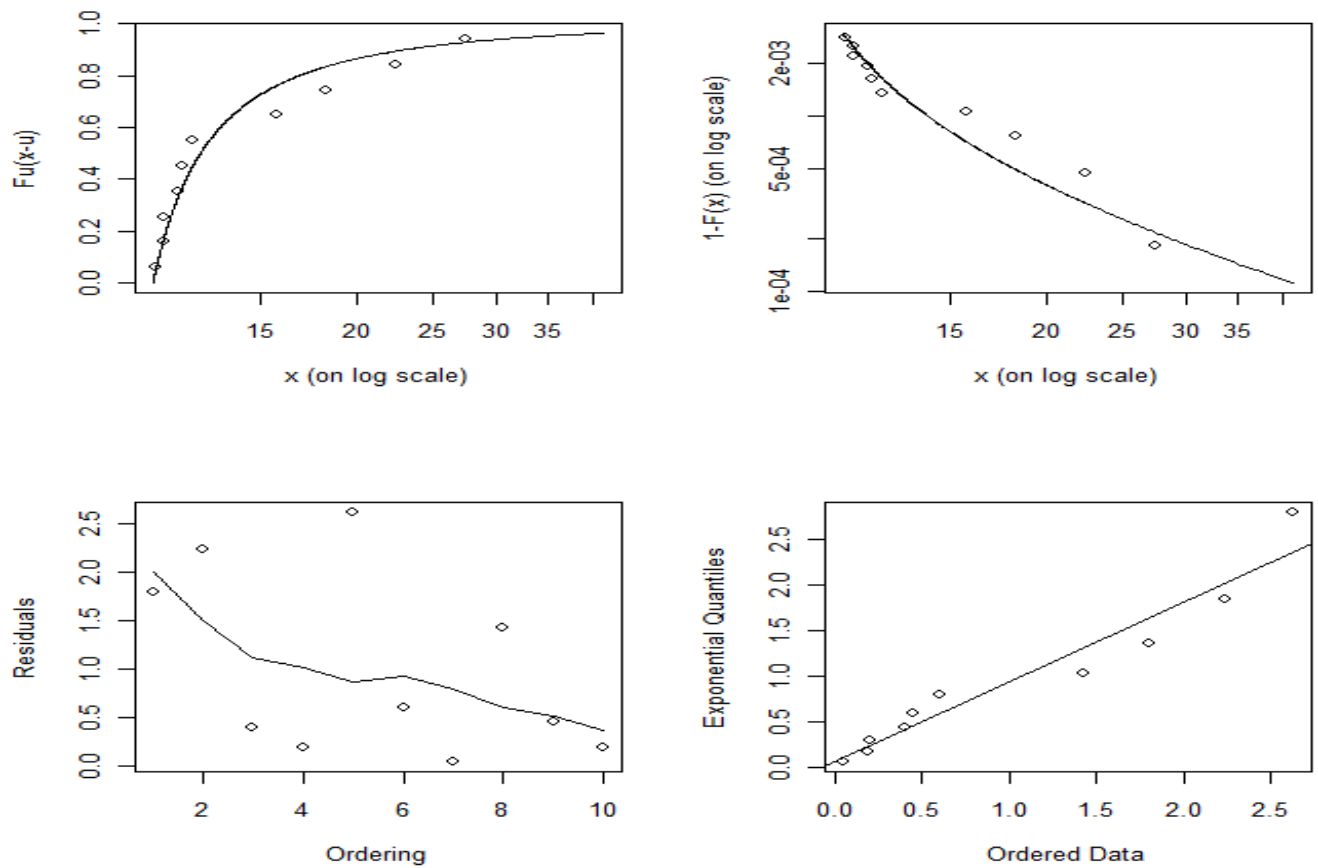**Figure 7.** Diagnostic plots for infrastructure.
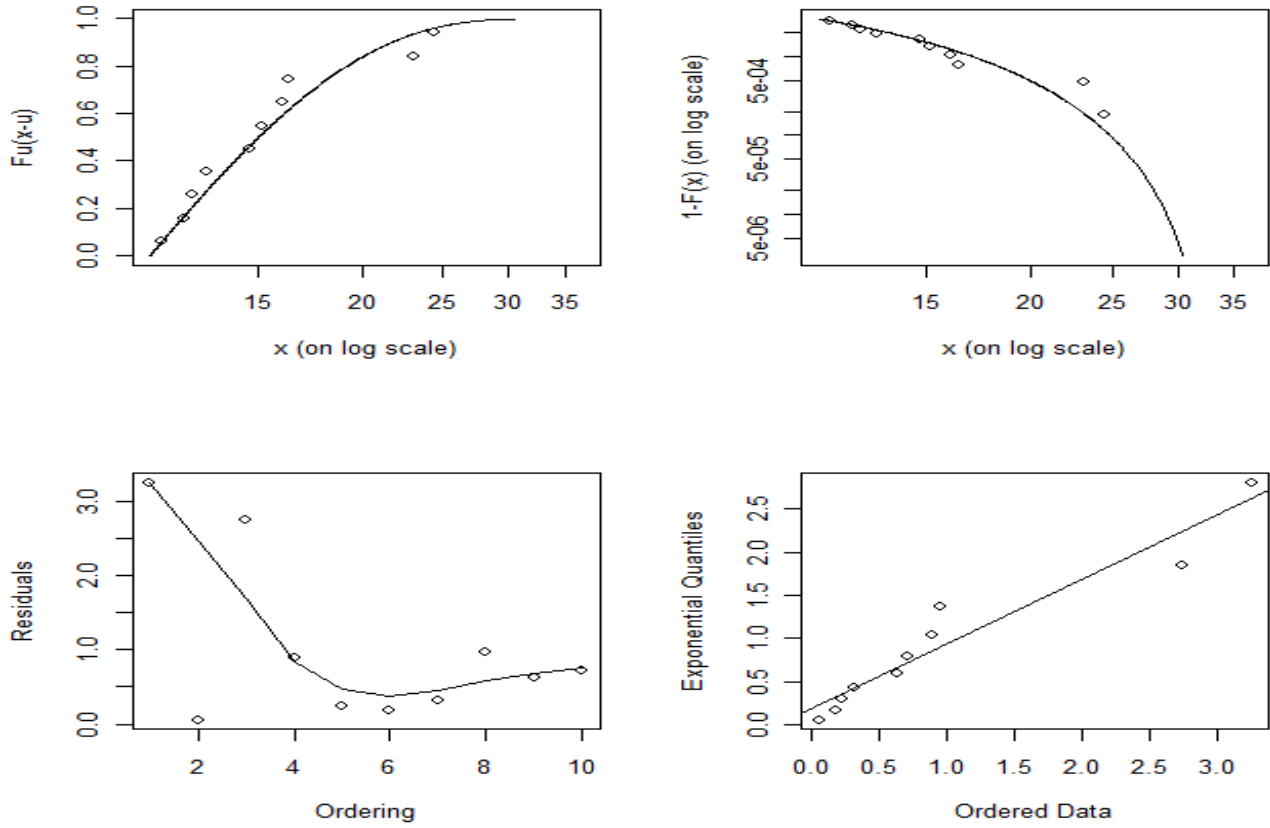

Figure 8. Diagnostic plots for construction sector.

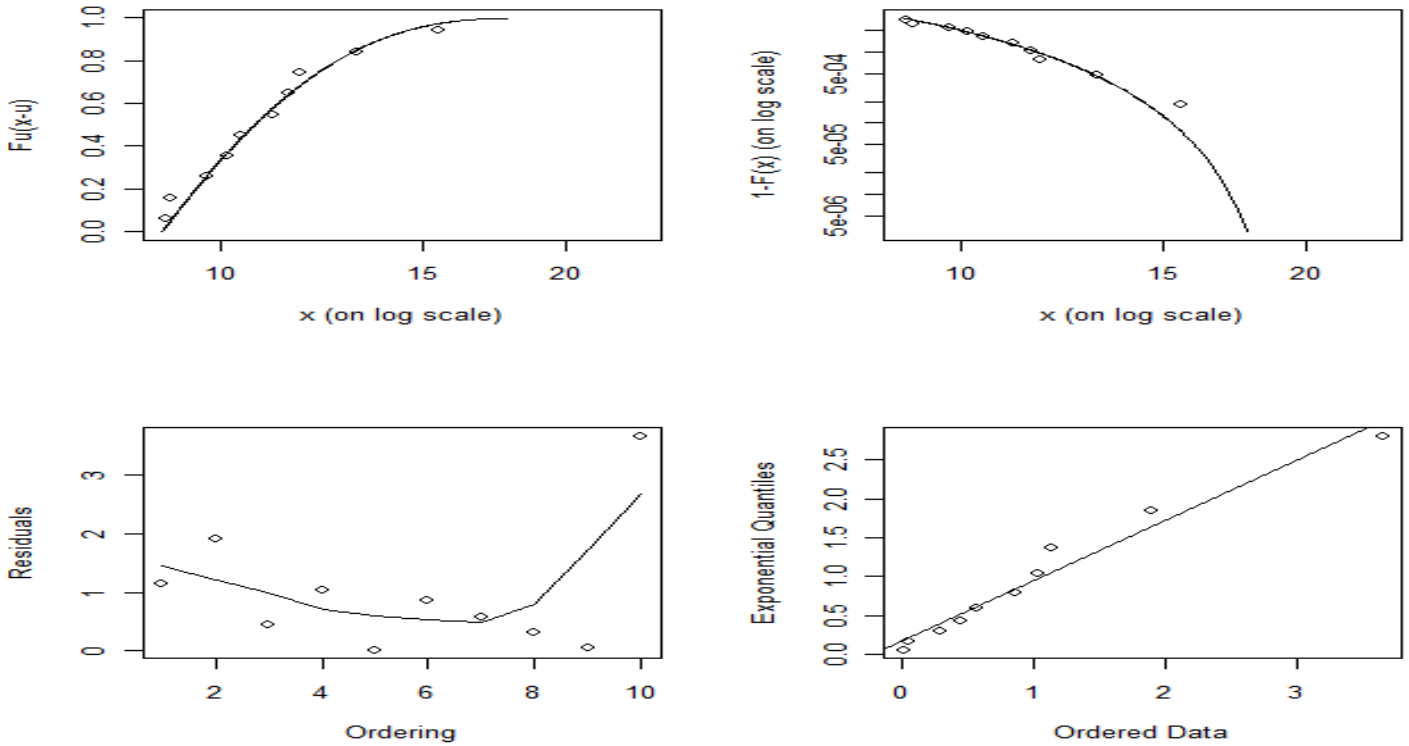**Figure 9.** Diagnostic plots for technology sector.

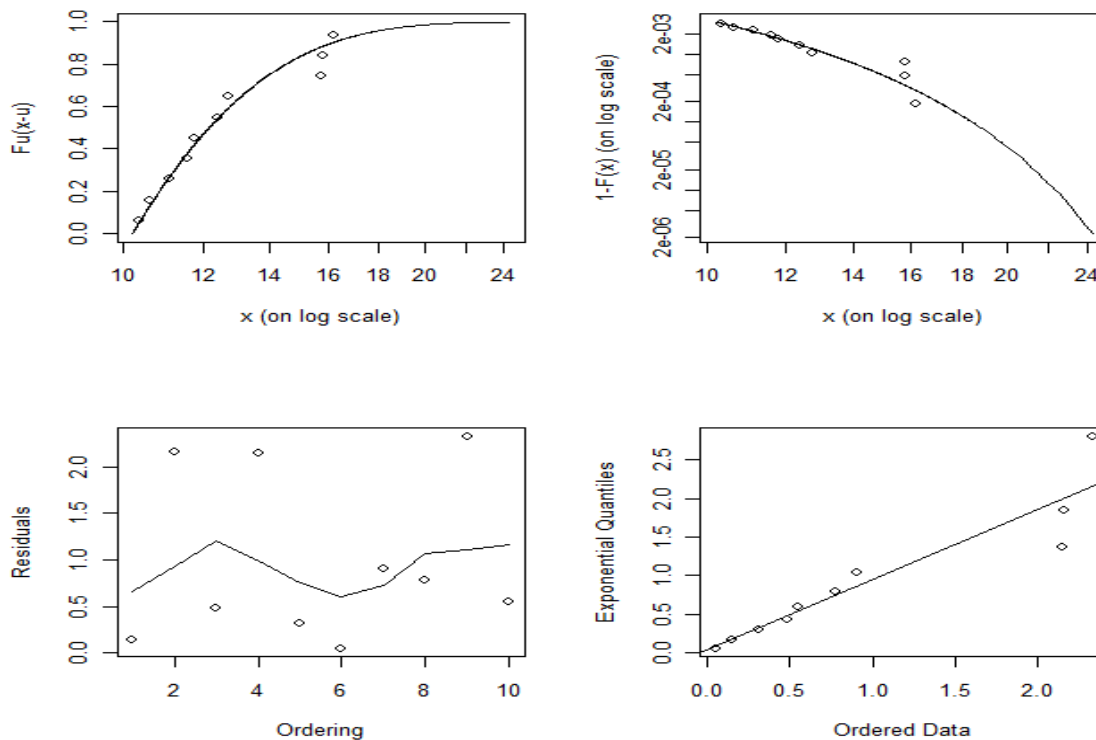**Figure 10.** Diagnostic plots for plantation sector.

Figure 11. Diagnostic plots for hotel sector.

finance, property, consumer-products, industrial-products, infrastructure, construction, technology, plantation and hotel.

This study demonstrates that the RiskMetrics under-estimate the risks of most sectors. In addition, the GPD, which is considered in the POT method, provides an adequate fit for the data. With the increased momentum in risk modelling brought by the Basel II Accord, and the relative lack of VaR and CVaR studies in Malaysia, there is a significant scope for additional studies on this topic, particularly with regards to CVaR in both financial market and credit risk.

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